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## SÉRIE DE DOCUMENTS DE RECHERCHE

**Impact of innovations in education: A theoretical approach under uncertainty**

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*Abstract:* In the context of the Millennium Development Goal of universal education, many developing countries have adapted their educational system. Various interventions and experiments have been implemented with the ultimate goal of improving the performance of pupils and schools. While lot of empirical studies have pointed mixed results of these experiments, the role of `uncertainty' following from the state of the nature on the education production function of school and households alongside the efficiency of these interventions still lack economic mechanism. This paper aims at developing a theoretical framework to link every specific intervention for each stakeholder to the global performance of pupils in education, taking into account the social welfare maximization problem.

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## 1. Introduction

Innovation is the application of a new resource or approach that changes practices and creates some value in education. The value can result to better scores for pupils and better performances of teachers. In 'Educational Epidemic' (2003), Roger Hargreaves sets that: the opportunity to engage actively in innovation and the means of transferring the validated innovation within schools and into other schools, are conditions that can support innovations in education.

The success of innovation in education depends on the capacity and disposition of the innovator as shown by Zhao et al (2002), Groff and Mouza (2008). The relationship between these two factors can be analyzed in two models:

- The Distance and Dependence model that makes explicit how the specific context can affect an innovation and can help identify its success by showing its difference from existing practice and resources.
- The Layers of Influence Model that distinguishes the influences that affect the conditions of an innovation.

Within the context of technology-based innovation the first model sheds a new light on the classical discussion on "pedagogy before practice" by suggesting that the realization of an innovation depends on a close connection of practice and technology issues. The model is initially drawn from a study on technology-based innovation, but it can also be used on other non-technology-based innovations in education. The model supports an understanding of educational innovations by depicting how an innovation can be assessed by its distance from current practice and its dependence on available resources. If the innovation is close to existing practice, it has better chances to be accepted. The practice can be related to classrooms, pedagogy, school structures, etc. An innovation in education needs also resources for success. This relates to the resources needed for the innovation and the extent to which they differ from existing school resources. These latter are technical resources (equipment), human resources (extra staff to support activities and planning time), or physical resources (classroom space). For example, if an innovation requires a great change in home inputs practice and more financial resources, then it needs more support to succeed than an innovation which requires fewer resources and that demands little change from the home's existing practice. As such, implementing an innovation in a school can imply that the school undergoes other innovations so as to give response to the need of resource and pedagogical demands implied by the initial innovation.

Innovation in education is also complex. The complexity can be analyzed at the national community, school or individual levels, and its success depends on the extent to which the change is understood and resourced. One lesson from this model is that it leads to better understand the different perceptions of the necessary requirements for an innovation to succeed. Fostering innovation in education needs to analyze barriers and resistances to an innovation within its context as it implies different views on the expected change. That is why there are many of attempts to understand the resistances to innovation in education. The school is a complex set of relationships and the implementation of an innovation, with its barriers and resistances can be approached from different levels. But attempting to understand the most effective ways of sharing innovations may be more useful than trying to overcome expected barriers.

The second model is called model of Layers of Influence. It analyzes the influences that affect an innovator's ability to realize and implement an innovation. It gives a way to conceptualize the layers of influence that affect the innovation and the innovator. In the literature four core layers are commonly identified and their influence analyzed:

- The innovation: The factors associated with the innovation itself. The way an innovation is perceived and its difference from existing practices are very important for its success. This latter requires an understanding of both the distance from current practice and the dependence on resources. The issue of the dissemination of an innovation is also crucial. The best dissemination requires longevity (sustainable over time), fecundity (applicable by different practitioners) and copy fidelity (replicable in local conditions) for an innovation.
- The micro level influences: The influence relevant to the innovator himself, such as his capacity and disposition to innovate.
- The mid-level influences: The local level influences such as school cultures, school management structures, and school infrastructure; and 'local' influences from the communities and the local authorities.
- The macro level influences: The government initiatives, national policy and programs.

This model highlights the way in which these layers traditionally interact and how the environmental conditions surrounding the innovation play.

*Supportive informal social environment:* It is very important for the success of innovations. Because a good atmosphere for innovation can help people to try or adopt new practices. For example, an atmosphere of competition in schools can enhance the efforts of innovation among local communities or even among teachers via their training practices

*Formal environment:* It's the organizational infrastructure of an educational system. This includes its formal policies and structures. This environment is crucial in providing resources for innovations and letting it be accessible through technical support and supportive access policies. For example food policies for pupils' better access to enough calories suppose the existence of canteen in schools.

The formal environment has a key role in creating spaces for sharing existing or innovative practices. It makes more easy, the partnerships of all stakeholders in education.

*Risk aversion:* Risk aversion is shown as an important element that inhibits the ability to innovate and it has implications on the extent to which any educational policy sets the appropriate conditions for innovation. Fear of failure in innovation implies a risk which can make stakeholders reluctant to innovate in education. Then, to overcome risk aversion, motivation to innovate must be created. A motivation can come from teachers wishing to improve the learning experience for pupils, or communities trying to improve competition between schools by rewarding initiatives. Local and national levels of motivations can be found in education with institutional practices that reduce risk aversion.

*Shared visions:* Common perceptions of the goals and requirements of an educative innovation are important. For example, teacher's perception on the effectiveness of new pedagogical practices implied by an innovation can influence the success of this latter. A shared vision for an innovation provides a crucial clarity of purpose and direction for those who manage innovations.

Throughout this discussion on the conditions and barriers of the success of educational innovation, it appears that enabling the good conditions for the effectiveness of innovation is not an easy goal. The specific layers of influence within which innovation is implemented play a great role, for the definition of educative policies and this is particularly important for developing countries, especially in Sub-Saharan Africa. The issue raised by innovation in these countries can be stated as follow: How does uncertainty affect the impact of innovations in education for developing countries? Indeed, in these countries, education becomes more and more crucial in development programs, and the stakeholders implied in education are

becoming better prepared to undertake new practices for education performance. For example, the rapid increase in enrollment rates and the budgetary constraints of these countries led to questions about the performance of interventions in the educational environment, in terms of academic achievement. The empirical analysis of performance consists of measures of pupils learning at two points: first, at the beginning of the school year (before the experiment), to measure initial skills, and then at the end of the school year (or after the experiment). This allows an evaluation at the end of the year which may be qualified by the skills of the child measured earlier that year, and an output measure of school can be obtained, depending on the pedagogical means that the pupil has accessed. Pupils are often evaluated in the basic disciplines of primary education. At first glance it may seem easy, especially in the case of primary education, to describe the school organization and the interactions it has on pupils' achievement. To measure the production of educational service, one identifies factors that influence learning achievements. It seems necessary to standardize the assessment in its content and its mode of administration. The priority is not to know the knowledge of a pupil, but why beyond such a class, a pupil learns better than another.

The key variables that affect performance are the context (as the group size, the pedagogical skills of teachers), the availability and accessibility of infrastructure and teaching materials, etc. Nevertheless, studies on education are very interested in the issue of allocative efficiency of resources related to education, and effective intervention could be reduced to a matter of optimizing the allocation of these resources.

Technical advances in the evaluation of school performance, through the definition of reliable indicators and the availability of survey data on acquisitions of scholars showed that the problem of effectiveness of interventions depends on both measuring the performance of schools and of individuals. Thus, whatever the technical and financial resources injected in schools, performance is largely dependent on aspirations placed by the society in education, and rules and standards that guide the school. The context of performance expected from pupils and schools is conditioned on the organization of education systems, and also on the demand and supply of education (Pritchett, 2001). This explains why in developing countries, it is difficult to predict the individual profitability of educational investment, which depends largely on the quality of learning.

This paper aims to develop a set of theoretical frameworks to link every specific type of new intervention for each stakeholder to the global performance of education, taking into account

the social welfare maximization problem. In section 2, we present the identification of the educational production performance. In section 3 the issue of interventions in education is treated and finally in section 4 we introduce the optimality of innovations and social welfare.

## **2. The identification of the educational production performance**

The goal of achieving universal education in developing countries involves looking for ways to produce effective and efficient school. Effective teaching methods, based on survey data acquisition school, have shown their worth for almost fifteen years. In order to identify forms of effective schools, tools were developed primarily to measure whether countries can achieve the goal to receive all the children of school age, and then to evaluate the effectiveness and quality of learning provided in schools.

Thus, since the 1990s, PASEC realized in Francophone Africa, surveys to assess child learning, collecting information on their characteristics: origin of children, their living environments, characteristics of teachers and schools, etc. These elements are often used as components of a production function of school (Bourdon, 2005). The problem is whether there is a form of this function that is appropriate to describe the effective provision of universal education and the performance of interventions in the education sector. A key issue relies in describing the cost of education.

### **2.1 The production function of school**

As pointed out by Coleman (1966), the identification of the determinants of educational service quality is not trivial. Hanushek (1986) showed that there is a bewildering range of issues including technical and esoteric conflicting results on the production process of the schools. He argues that there is still no answer to the question: what are the factors that influence pupils' performance? One answer is given by Filmer and Pritchett (1999), with an economic approach. These authors find that the choices that guide an educational allowance are often not based on academic performance. Empirical facts contradict the hypothesis of an efficient allocation of resources that seeks to maximize the school performance. There are four reasons. First, the school is not a black box which production technology follows market rules. Then, the impact of schooling on the attainment may be small compared to the role and importance of innate abilities of learners. In addition, the demand for education is not facing a market, and the production function can be observed not an effective economic standpoint.

Finally, the education production function, if it is tested econometrically, cannot be generalized as already shown by Hanushek (1986).

## **2.2 Methods for the measurement of educational effectiveness: A brief review**

The optimal combination of school time programs has been the subject of studies developed by Charnes, Cooper and Rhodes (1978) who were inspired by the contribution of Farrell (1957). But the difficulty lies in identifying stable parameters in the production function. School factors are considered outside the school: social class of the household, parental education level, time spent by parents in school support. The school production is represented by the results of pupil assessment in language and calculation, and the value of self-esteem reported by the pupil. The study of the effectiveness of the education sector is often based on the approach panel of Battese and Coelli (1995). The authors showed that in terms of stochastic fixed effects of environmental variables can explain the remoteness of the border. Empirical studies are also interested in identifying the best performing schools. Relying on parametric and nonparametric approaches, Cooper and Cohn (1997) applied showed the type of schools that are close to the efficiency frontier, using envelope method. They followed more than 540 primary schools. Output variables describe the class average standardized tests in math and reading. The input variables are of two groups: teacher characteristics and average characteristics of the teaching group (size, gender and ethnic compositions of eligible children for social programs). The impacts of intervention methods on the effectiveness of school have been examined as well by Stiefel, Rubenstein and Schwartz (1999). The authors showed that there are strong inertia between interventions and their effects on academic performance.

In general, the output is measured as an average performance in school tests and the factors are the resources allocated to the education system. Klein (2007) used a Becker-Stigler-Peltzman-like model to determine the socially optimal level of intervention in education.

Other studies have tried to link school performance to the time of enrollment. Thus, for higher education Dolton, Marcenaro and Navarro (2003) described a production function where academic success, given by individual performance on the final exam, depends on the time spent at school. They showed that the study time is four times less profitable than teaching in a group work.

As general form of intervention, there is public expenditure on education, whose effectiveness has been investigated. Gupta and Verhoeven (2001) evaluated the effectiveness of public

expenditures in 37 African countries over the period 1984-1995, and compared them with Asian and Western countries. They showed that on average, African countries are less efficient than Asian and those in the Western Hemisphere.

Afonso, Schuknecht and Tanzi (2006) showed a clear distinction between countries according to indicators of absolute performance and cost effective type indicators. National structures for utility costs can play a crucial role and lead to that some systems offer public service and others do not have incentives to seek, because of allocation rules, routine border performance allowed by the technical frontier.

Kirjavainen and Loikkanen (1998) used a Tobit model powered by the levels of efficiency from the DEA analysis to explain the determinants of efficiency of secondary schools in Finland. In their approach, the educational level of parents plays a key role, as a factor explaining differences in school performance. Bradley, Johnes and Millington (2001) also used DEA and Tobit model to evaluate the technical efficiency of English secondary schools. The efficiency rate means obtained are between 83 and 75%. These authors found that competition between schools improves the efficiency level of schools.

The latter result is confirmed by Waldo (2001) who measured the performance of Swedish secondary schools using DEA. In the case of the Portuguese side, Oliveira and Santos (2005) examined institution indicators. They authors were particularly interested in releasing the convexity constraint. In specific cases, a two step approach is used, where the environmental variables are preferred, with the application of bootstrapping (Simar and Wilson, 2003). The study concluded that the unemployment rate, access to health services, adult education and infrastructure endowments are determinants of academic performance. Rubenstein et al. (2007) studied the consistency of efficacy at two levels: the school and the school district. In a sample of schools in the north-eastern United States, it showed that even if policies favoring areas are implemented, their impact is conditioned by structural elements including a key based on time vocational training, through practice, etc.

This brief literature review confirms the ambiguity and difficulty of measuring the efficiency and performance. The school performance is one that has better results, but it can also be one that is in a very favorable environment. It is better to correct the academic performance of the impact of context. For this, the score of a pupil in the same context in each school, can be estimated. This is called "adjusted score". It is the score of a pupil adjusted for the impact that would have obtained the average pupil in each class. This is the main finding of the literature

on school effectiveness. The link in practice was shown for secondary schools in Spain by Muniz (2002).

### **3. Issue of interventions in education**

Education is largely a national public service, whose organization and operation are provided by the government. However, local administration can also be involved in the development of this public service. There are several stakeholders in the education sector, each with specific and complementary roles.

*The Government:* It is competent in all aspects of pedagogy, curricula, national qualifications, the management of teaching staff, etc. Staffs working in departments of education are central government, distinct from decentralized ministry present in the academies and departments.

*Local administration:* Administration of Education is also at the regional and district level: it is the decentralized services of the ministries of education. The municipality, the county and the region participate in the functioning of national education.

*Parents of children:* They parents are full members of the educational community. They are fully involved in the education of their children and play direct roles. Through their representatives, they participate in school councils, class, and administration of the institutions which indirectly implies the application of educational policy.

It should be noted that there are often structures of consultation (which give their opinion to guide decision-making or allow actors and partners of education to meet and take all decisions) and sometimes technical committees dealing with issues of collective interest.

Interventions of all these stakeholders in education have direct and indirect impacts on pupils' performance. They are of various kinds: those made by the Government (in health or nutrition, education, supply, management and others), and by households (greater parental involvement, tutoring for children, shopping new materials, etc.).

*The issue raised can be stated as follow: How uncertainty does affect the impact of innovations in education?*

### 3.1 The benchmark model without uncertainty

The performances of a child are measured by the scores on a set of tests.  $X_t$  denote a vector whose  $n$  components are the criteria measuring pupils' performance. All innovations are captured by the vector  $U_t$  with  $r$  components. The aim is to start from an initial situation  $X_0$  to an optimal situation  $X_T$ , where pupils' performance is better.  $T$  is the final time for the effects of innovations. The equation of variation in pupils' performance, thanks to innovations (in the absence of uncertainty and externalities) is:

$$\dot{X}_t = F(t, X_t, U_t) \quad (1)$$

The optimality means that in the last situation, the interventions lead to a state near to their objectives. At the level of a pupil, it does not mean that all his scores at the period  $T$  are higher than in the initial period. But the average level of scores achieved with  $X_T$  is expected to be higher than with  $X_0$ .

#### a) The linear case

First, we can take a simple case, the linear form as follows:

$$\dot{X}_t = P(t)X_t + Q(t)U_t + R(t) \quad (2)$$

The elements of the matrices  $P(t)$  and  $Q(t)$ , of respective orders  $n \times n$  and  $n \times r$  are defined in time. For the achievement of performance to the final time  $T$ , consider the vectors  $X_0$  and  $X_T$  representing the pupils' performance, at the beginning and at the end of the period.

At this step, we can use different cases: independence of innovations and existence of a centralized public (as a global educational policy overseen by the Government, in the form of recommendations to stakeholders and sharing of innovations) that guide these interventions. We will treat every one of these cases.

**Proposition 1:** The interventions leading to the vector  $X_0$  to  $X_T$  from the beginning and the end of the period, can be written as:  $U = B * c + v$  (3)

Where we have:  $c = A(T)[Y^{-1}(T)X_T - \int_0^T Y^{-1}(\theta)R(\theta)d\theta - X_0]$  is a constant vector,  $v$  is a function of time and  $B$  is a vector.  $Y(t)$  is the fundamental matrix of the system:  $\dot{X}_t = P(t)X_t$

and  $B(t) = Y^{-1}(t)Q(T)$ . We will define their expressions. The economic interpretation of this expression is that at any point of time, it is possible to have optimal actions guiding the child performances.

Proof: Let  $Y(t)$  be the fundamental matrix of the system:  $\dot{X}_t = P(t)X_t$  with the initial condition:  $Y(0) = E$ .  $E$  is the unity matrix. If we replace the vector  $X$  by  $Y(t)Z$  where  $Z$  is another unknown function. We have :

$$\dot{Z}_t = B(t)U_t + G(t) \quad (4)$$

Where  $B(t) = Y^{-1}(t)Q(t)$  and  $G(t) = Y^{-1}(t)R(t)$

By making the following transformation:

$$Z = \xi + \int_0^t G(\theta)d\theta \quad (5)$$

The vectorial function  $\xi$  verifies the relation:

$$\dot{\xi}_t = B(t)U_t \quad (6)$$

With these transformations, we then have:

$$\begin{cases} \xi(0) = \xi_0 = X_0 \\ \xi(T) = \xi_T = Y^{-1}(T)X_T - \int_0^T Y^{-1}(\theta)R(\theta)d\theta \end{cases}$$

For sake of simplicity, let us suppose that  $r = 1$ , meaning that we have only one intervention.

The integration of (6) gives the following system of integral equations:

$$\xi_T - \xi_0 = \int_0^T B(\theta)U(\theta)d\theta \quad (7)$$

The solution is to be of the form:  $U = B * c + v$  (8)

The vector  $c$  is a constant that is to be defined and  $v$  is a function such that:

$$\int_0^T b_s(\theta)v(\theta)d\theta = 0, s = 1, 2, 3, \dots, n \quad (9)$$

The  $b_s$  are the components of the vector  $B$

### b) The non-linear case

We also deal with this case:

$$\dot{X}_t = P(t)X_t + Q(t)U_t + R(t) + \mu G(t, X_t, U_t, \mu) \quad (10)$$

G is a vector function and a parameter  $\mu$ .

Case of impulse innovations: the innovations are constant, that is to say the same kind, between time  $t_0$  (initial) and the time T. Specifically, the vector  $U_t$  is constant in time intervals  $[t_j, t_{j+1}]$ , bounds,  $t_0, t_1, t_2, \dots T$ . This is the case of school feeding, for example, where foods are provided at fixed hours, identical, to the time parents spend for helping their children, regularly paid scholarships, etc.

**Proposition 2:** We associate to every couple of bounded sets  $G_0$  and  $G_T$  a scalar  $\mu_0 = \mu_0(G_0, G_T)$  such, whatever  $\mu_0 < \mu_T$ , there exists an intervention  $U_t$  that leads the vector  $X_0$  from  $G_0$  to  $X_T$  belonging to  $G_T$ .

Proof : we set  $A(T) = \int_0^T BB^* dt$  where  $B(t) = Y^l(t)Q(t)$

Suppose that we have X and U such that the equation of the non linear case is verified. The equation can be written as :

$$X(t) = Y(t)X_0 + Y(t) \int_0^t Y^{-1}(\theta)[Q(\theta)U(\theta) + R(\theta) + \mu G(\theta, X, U, \mu)]d\theta \quad (11)$$

By setting  $t = T$ , we have:

$$Y^{-1}(T)X_T - X_0 = \int_0^T B(\theta)U(\theta)d\theta + \int_0^T Y^{-1}(\theta)[Q(\theta)U(\theta) + R(\theta) + \mu G(\theta, X, U, \mu)]d\theta \quad (12)$$

Writing  $U = B * c + v$ , we have :

$$c = A(T)[Y^{-1}(T)X_T - X_0 - \int_0^T Y^{-1}(\theta)R(\theta)d\theta - \mu \int_0^T Y^{-1}(\theta)G(\theta, X, U, \mu)d\theta] \quad (13)$$

Putting this expression of c in that of U, we have:

$$U(t) = U_0(t) + \mu B^* A^{-1}(T)[-\int_0^T Y^{-1}(\theta)G(\theta, X, U, \mu)]d\theta \quad (14)$$

$$U_0(t) = B^* A^{-1}(T) \left[ Y^{-1}(T)X_T - X_0 - \int_0^T Y^{-1}(\theta)R(\theta)d\theta \right] + v(t) \quad (15)$$

It is easy to find that

$$X(t) = X_0(t) + \mu \left[ \int_0^T Y(\theta) Y^{-1}(\theta) G(\theta, X, U, \mu) d\theta - Y(t) A(t) A^{-1}(T) \int_0^T Y^{-1}(\theta) G(\theta, X, U, \mu) d\theta \right] \quad (16)$$

$$\text{With } X_0(t) = Y(t) \left[ X_0 + \int_0^t B(\theta) U_0(\theta) d\theta + \int_0^t Y^{-1}(\theta) R(\theta) d\theta \right] \quad (17)$$

The equations (15) and (17) show that, for  $\mu = 0$  and  $U = U_0(t)$  the equation (10) has  $X = X_0(t)$  as solution.

Some simple examples can help understand the way these cases can be used.

**Example 1 :** Government intervention: Suppose that either an experimental or expenditure on food for the children and we measure the effects of their time concentrating in class. This time of concentration is assumed to have a direct impact on student scores.

$$\dot{X}_t = AX_t + GU_t \quad (18)$$

Variable X measures the difference between the maximum score and its current level.

$$X_t = \begin{bmatrix} X_1 \\ \dots \\ X_n \end{bmatrix} \quad A = \begin{bmatrix} A_{11} & \dots & A_{1n} \\ \dots & \dots & \dots \\ A_{n1} & \dots & A_{nn} \end{bmatrix} \quad G = \begin{bmatrix} G_{11} & \dots & G_{1n} \\ \dots & \dots & \dots \\ G_{n1} & \dots & A_{nr} \end{bmatrix} \text{ et}$$

$$U_t = \begin{bmatrix} U_1 \\ \dots \\ U_r \end{bmatrix}$$

The vector equation is equivalent to following the scalar equation:

$$\frac{dX_j}{dt} = \sum_{k=1}^n A_{jk} X_k + \sum_{l=1}^r G_{jl} U_l \quad \text{avec } j = 1, \dots, n$$

The solution of the equation is :

$$X_t = X_0 e^{At} + \int_0^t e^{A(t-\theta)} GU(\theta) d\theta \quad (19)$$

$U_t$  interventions can reduce scores gaps at time  $t = T$  to the state  $X_T = 0$ . One can show that innovation can write U in the form:

$$\tilde{U}_t = -G^* e^{A^*t} M^{-1} X_0 \quad (20)$$

Where

$$M = \int_0^t e^{-A\theta} G G^* e^{-A^*\theta} d\theta \quad (21)$$

The asterisk is the transposed matrix. By taking the following simple example:

$$\dot{X}_t = a + U_t \quad (22)$$

The innovation becomes:

$$\tilde{U} = -\frac{2a}{1-e^{-2aT}} e^{-at} X_0 \quad (23)$$

### Example 2: Community intervention

We can think in discrete time. The following equation describes the change in the score over time:

$$X_t - X_{t-1} = aX_{t-1} + bU_t + r \quad (24)$$

The equation would be interpreted as follows: the difference between the scores before and after the access to the canteen, depends on initial score, and intervention U. For the intervention is optimal, in the sense that  $X_t > X_{t-1}$ , it must be of the form  $U = B * c + v$ , that is to say, it is constant in the time interval  $[t-1, t]$ . The recurrence gives:

$$X_t = (1 + a)^t X_0 + \sum_{j=1}^t (1 + a)^{j-1} (b U_{n-j+1} + r) \quad (25)$$

This equation gives results based interventions  $U_i, i \leq t$ .

As long as the intervention is constant over time, we have:

$$U_1 = \frac{X_t - (1 + a)^t X_0}{b \sum_{j=1}^t (1 + a)^{j-1}} - \frac{r}{b}$$

**Example 3:** intervention of local authorities: we consider a school located in an area where local government tries to raise the level of academic achievement. The intervention is to

support either a given number of pupils.  $X$  can be the number of pupils who achieve the primary school and  $X_0$  is the first pupils enrolled.  $U$  is the proportion of pupils involved in a school achievement program. Suppose that the decision depends on the following expression:

$$\dot{X}_t = -aX + bU$$

The intervention is restricted to the following constraints:  $0 \leq U \leq 1$

The aim is to set the final value  $X_T$  as large as possible, at the end of the process. It is easy to find that  $U = 1$  and  $\max_{-1 \leq U \leq 1} X_T = X_0 e^{-aT} + \frac{b}{a}(1 - e^{-aT})$

### 3.2 Impacts of innovations under uncertainty

As discussed in the introduction, innovation on education depends on many uncontrolled factor that may impact its effectiveness. Informal and formal environments, risk aversion, a common vision and other parameters have to be taken into account. We suppose that they influence the final goal of an innovation, leading to this kind of equations of motion guiding the actions of the interveners are:

$$\dot{X}_t = A(t)X_t + B(t)U_t + R(t) + C(t)Y_t \quad (26)$$

$A_t$ ,  $B_t$ ,  $C_t$  and  $R(t)$  are vectors whose components depend on the time and they are defined in the interval  $[0, T]$ . The criteria measuring pupils' performance is always the vector  $X_t$ . All innovations remain captured by the vector  $U_t$ .

$Y_t$  is a vector with probabilistic components:  $y_1(t), y_2(t), \dots, y_m(t)$ . They are also defined in the interval  $[0, T]$ . We note by  $Z(t)$  the fundamental matrix of the homogeneous system:

$\dot{X}_t = A(t)X_t$  with  $Z(0) = E$ , where  $E$  is the unity matrix. Suppose that  $D$  is a domain in the phase space and define:

$$J(U_t) = Prob\{X(T) \in D\} \quad (27)$$

The functional  $J$  is the probability that the end of the stochastic trajectory of the performances arrives at the region  $D$ . The issue is to find the better interventions,  $U_t$  that lead to a maximum level of  $J$ .

We can note that the mathematical expectation of  $X_t$  is:

$$E[X_t] = Z(t)E[X_0] + \int_0^t Z(\theta)Z^{-1}(\theta)[BU_\theta + R(\theta) + CE[Y]]d\theta \quad (28)$$

This expression leads to simple forms of the variance of  $X_t$ , depending on the components  $a_s$  ( $s = 1, 2, \dots, n$ ) of  $E[X_t]$ .

We have for each component  $a_s$ , with:  $s = 1, 2, \dots, n$

$$a_s = a_s^0(t) + \int_0^t \sum_{j=1}^r c_{sj}(t, \theta) U_j(\theta) d\theta \quad (29)$$

The components of the variance of  $X_t$  are:

$$V_s(t) = E[X_s(t) - a_s(t)]^2 \quad (30)$$

With  $s = 1, 2, \dots, n$

#### 4. Optimality of innovations and social welfare

The two propositions define the existence of interventions, according to the form of the variations of the performances. We can now extend our approach by using the concept of social welfare that integrates utility functions of all stakeholders in educations.

We define this form of social welfare function:

$$W = \int L(S(p)) w(p) dp \quad (31)$$

$L$  is a utility function is defined on a set of variables including consumption in the household, health of the children. The function  $L$  is weighted by  $w(p)$  a function defined on the variable  $p$ , which is the rank of individuals in the population.  $S$  is a set of economic variables such as consumption, health, etc. The main issue is that many interveners can have different utility function they maximize. According to each innovator, we can define an explicit form of this function and aggregate them in a specific social welfare function. For this aim, the optimization problem can be stated as:

Starting from an initial situation  $X_0$  to an optimal situation  $X_T$ , where child performance is better, and the equation of variation in child's performance is,  $\dot{X}_t = F(t, X_t, U_t)$ , how can we find interventions leading to a maximum social welfare?

Our main issue is how to link every specific intervention for each stakeholder, to the global performance of pupils in education.

## 5. Conclusion

The impact of innovations in education is treated here by a theoretical approach. Various interventions and experiments have been implemented with the ultimate goal of improving the performance of pupils and schools. While lot of empirical studies have pointed mixed results of these experiments, the role of 'uncertainty' following from the state of the nature on the education production function of school and households alongside the efficiency of these interventions still lack economic mechanism. This paper develops theoretical frameworks linking every specific intervention to the stakeholders to the global performance of education system. After showing that innovation in education depends on many factors including the capacity and disposition of the innovator, the specific context and the layers of influence, we identify the educational production performance. The issues related of interventions in education are studied and we link the interventions to the tools that can be used aiming optimal impacts. Linear and non linear cases are also treated.

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